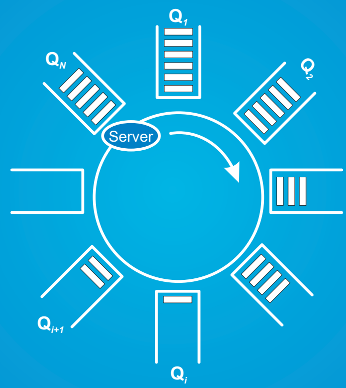
Polling Systems with Gated and Exhaustive cycles



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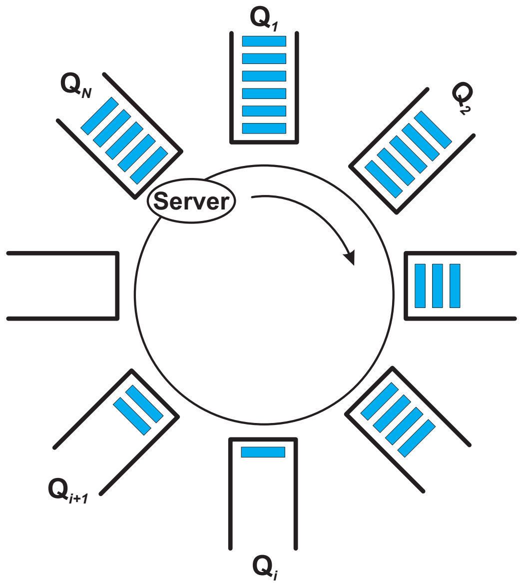
Introduction

Polling System

- N queues,
- Arrivals at Q_i: Poisson(λ_i),
- Service time at Q_i: B_i,
- Load at Q_i: $\rho_i = \lambda_i \mathbb{E}[B_i]$,
- Switch-over time to Q_i: S_i,
- Cycle time C_i starting from the beginning of a visit to Q_i.



$\mathbb{E}[W_i]$: mean waiting time at queue i.

Service disciplines

- *Gated discipline*: serve exactly those customers present upon arrival of the server

$$\mathbb{E}[W_i^{gated}] = (1+\rho_i)\mathbb{E}[R_{C_i}] \approx (1+\rho_i)\mathbb{E}[R_C]$$

- *Exhaustive discipline*: serve queue until empty

$$\mathbb{E}[W_i^{exh}] \approx (1-\rho_i)\mathbb{E}[R_C]$$

Model

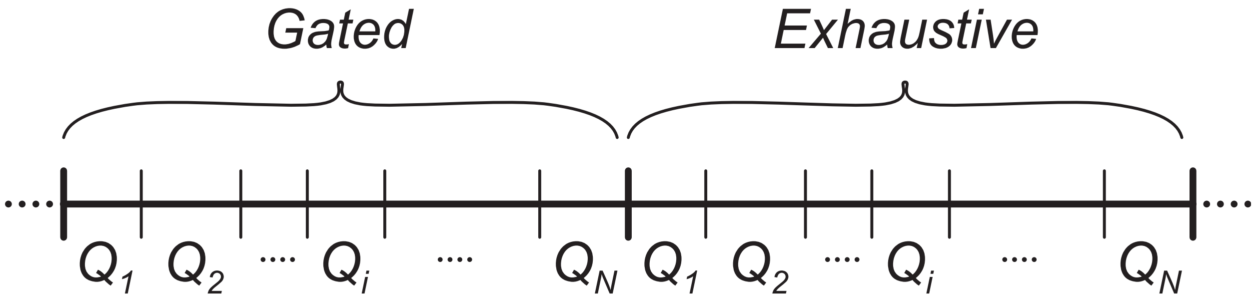
Gated/Exhaustive service discipline

$$\mathbb{E}[W_i^{gated}] \approx (1+\rho_i)\mathbb{E}[R_C]$$

$$\mathbb{E}[W_i^{exh}] \approx (1-\rho_i)\mathbb{E}[R_C]$$

$$\Downarrow$$

$$\mathbb{E}[W_i^{gat/exh}] \overset{??}{\approx} \mathbb{E}[R_C]$$



Model

Gated/Exhaustive service discipline

$$\mathbb{E}[W_i^{gated}] \approx (1+\rho_i)\mathbb{E}[R_C]$$
$$\mathbb{E}[W_i^{exh}] \approx (1-\rho_i)\mathbb{E}[R_C]$$
$$\Downarrow$$
$$\mathbb{E}[W_i^{gat/exh}] \overset{??}{\approx} \mathbb{E}[R_C]$$

Fairness: $\mathbb{E}[W_1] \approx \mathbb{E}[W_2] \approx \ldots \approx \mathbb{E}[W_N]$

Triggered by Park et al. (2005)

Model

Research Question

Gated/Exhaustive $\stackrel{??}{\Rightarrow} \mathbb{E}[W_1] \approx \mathbb{E}[W_2] \approx \ldots \approx \mathbb{E}[W_N]$??

Analyze Gated/Exhaustive discipline

Use Smart Customers

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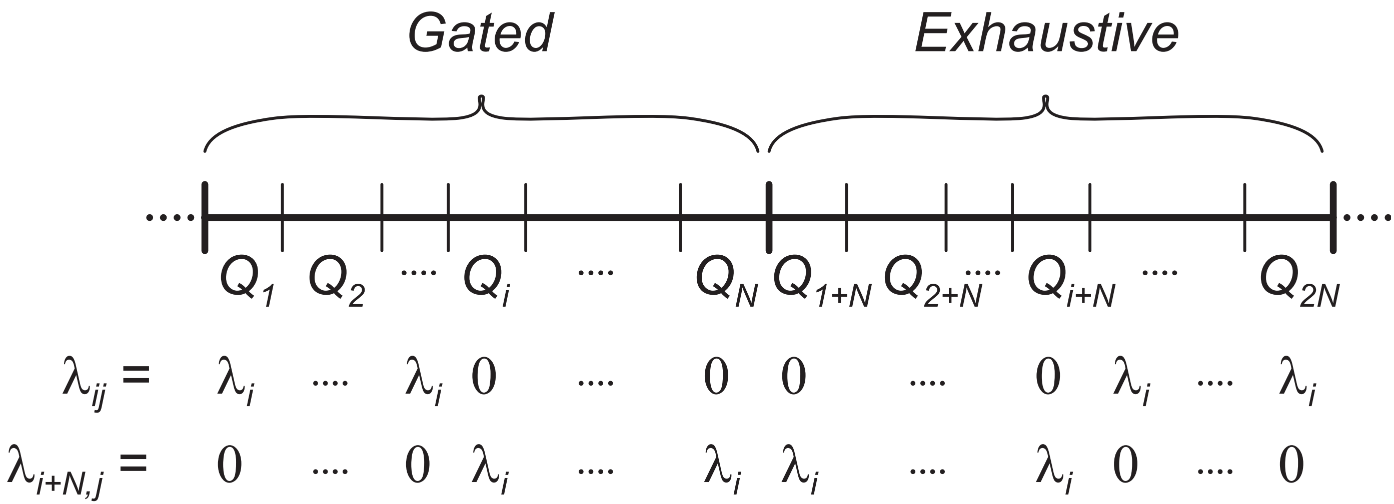
Smart Customers

Arrival rate at queue depends on position of the server:

- arrival rate λ_{ij} at Q_i while server working at Q_j



Analysis

Waiting times

Standard techniques adapted for Smart Customers yield waiting times

- Multitype Branching Processes
  $\rightarrow$ joint queue length & waiting time distributions

- Mean Value Analysis for Polling Models
  $\rightarrow$ $\mathbb{E}[W_i]$'s

Numerical results: Example for 2 queues

$$\lambda_1 = 0.6,\ \lambda_2 = 0.2,\ B_i, S_i \sim \exp(1)$$

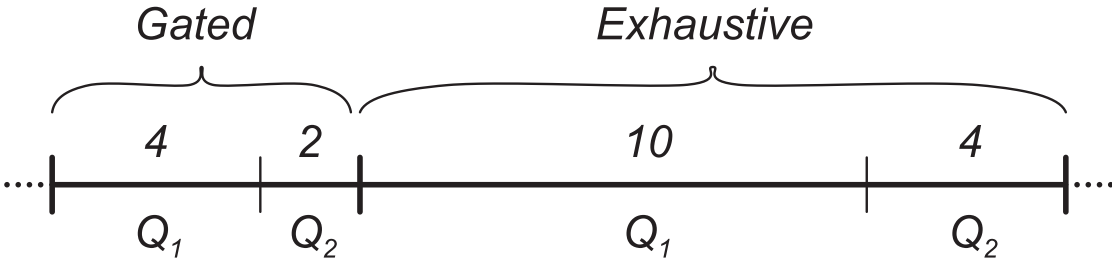
Strategy	$E[W_1]$	$E[W_2]$	$\lvert E[W_1] - E[W_2] \rvert$
Exh.	5.6	11.2	5.60
Gated	12.8	9.7	3.08
G/E (1)*	7.0	12.1	5.15
G/E (2)*	6.8	12.5	5.63

*start cycle at Q_1, respectively Q_2

Numerical results

Why does G/E not give roughly equal mean waiting times?

Mean cycle times gated and exhaustive are not equal:



Although *both for gated and exhaustive* $\mathbb{E}[C] = \frac{\mathbb{E}[S]}{1-\rho} = 10$:

Analysis

New Idea

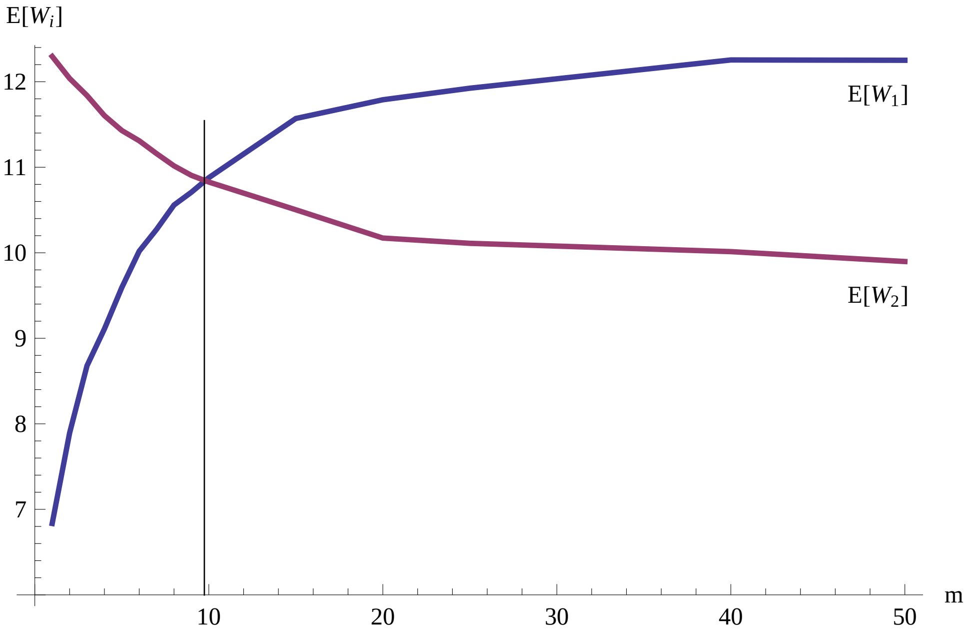
Instead of Gated-Exhaustive:

$\underbrace{\text{Gated - Gated - ...- Gated}}_{m\times}$ - Exhaustive

Analysis in the same way

Same example

$$\lambda_1 = 0.6,\ \lambda_2 = 0.2,\ B_i, S_i \sim \exp(1)$$



Conclusion

Summary

m-Gated/Exhaustive:

- 1-G/E : $\mathbb{E}[W_1] \neq \mathbb{E}[W_2]$
- m-G/E : $\mathbb{E}[W_1] \approx \mathbb{E}[W_2]$ (for appropriate m)

Research in progress

- Mixing $m_G\times$ Gated and $m_E\times$ Exhaustive
- With probability p cycle (or queue) Gated, otherwise Exhaustive